Date Planned : __ / __ / __	Daily Tutorial Sheet - 6	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	JEE Advanced (Archive)	Exact Duration :________

51. For any real number x, let $[x]$ denotes the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by $f(x)=\begin{cases} x-[x] \;\;, & if\;[x]\;is\;odd \\ 1+[x]-x, & if\;[x]\;is\;even \end{cases}$. Then, the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is: **(2010)**

52. Let $f:[0,2]\to R$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with $f(0)=1$.

Let $F(x)=\int_0^{x^2} f\left(\sqrt{t}\right)dt,\; for\; x\in[0,2]$. If $F'(x)=f'(x), \forall\, x\in(0,2)$, then F(2) equals:

(A) e^2-1 **(B)** e^4-1 **(C)** $e-1$ **(D)** e^4 **(2014)**

53. Let f be a non-negative function defined on the interval $[0,1]$. **(2009)**

If $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt=\int_0^x f(t)\,dt,\; 0\le x\le 1\; and\; f(0)=0$, then:

(A) $f\left(\frac{1}{2}\right)<\frac{1}{2}\, and\; f\left(\frac{1}{3}\right)>\frac{1}{3}$ **(B)** $f\left(\frac{1}{2}\right)>\frac{1}{2}\, and\; f\left(\frac{1}{3}\right)>\frac{1}{3}$

(C) $f\left(\frac{1}{2}\right)<\frac{1}{2}\, and\; f\left(\frac{1}{3}\right)<\frac{1}{3}$ **(D)** $f\left(\frac{1}{2}\right)>\frac{1}{2}\, and\; f\left(\frac{1}{3}\right)<\frac{1}{3}$

54. $\int_0^x f(t)\,dt = x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is: **(1998)**

(A) $\frac{1}{2}$ **(B)** 0 **(C)** 1 **(D)** $-\frac{1}{2}$

55. Let $f:R\to R$ be a differentiable function and $f(1)=4$. Then, the value of $\lim_{x\to 1}\int_4^{f(x)}\frac{2t}{x-1}dt$ is:

(A) $8f'(1)$ **(B)** $4f'(1)$ **(C)** $2f'(1)$ **(D)** $f'(1)$ **(1990)**

Paragraph for Questions 56 to 57

Let $f(x) = (1-x)^2 \sin^2 x + x^2, \forall x \in R$ and $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t)\, dt \;\; \forall x \in (1, \infty)$ **(2013)**

56. Consider the statements:

P: There exists some $x \in R$ such that, $f(x) + 2x = 2(1+x^2)$.

Q: There exists some $x \in R$ such that $2f(x) + 1 = 2x(1+x)$.

Then,

(A) both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) both P and Q are false

57. Which of the following is true?

(A) g is increasing on $(1, \infty)$
(B) g is decreasing on $(1, \infty)$
(C) g is increasing on (1, 2) and decreasing on $(2, \infty)$
(D) g is decreasing on (1, 2) and increasing on $(2, \infty)$

58. $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 + \cot x\, cosec\, x \\ \cos^2 x & \cos^2 x & cosec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$. Then $\int_0^{\pi/2} f(x)\, dx = \ldots$ **(1987)**

59. For $x > 0$, let $f(x) = \int_1^x \frac{\ln t}{1+t} dt$. Find the function $f(x) + f(1/x)$ and show that $f(e) + f(1/e) = 1/2$, where $\ln t = \log_e t$. **(2000)**

60. Let $a + b = 4$, where $a < 2$ and let $g(x)$ be a differentiable function. If $\frac{dg}{dx} > 0, \forall x$ prove that $\int_0^a g(x)dx + \int_0^b g(x)\, dx$ increasing as $(b-a)$ increases. **(1997)**